Low dimensional topology and number theory XIII
To the memory of Professor Toshie Takata

IMI Auditorium (413 Rm), 4F West 1st Bd, Kyushu University (Ito Campus)
15th March, 2022 ∼ 18th March, 2022

Program

March 15th (Tues)

9:40 Organizer

9:45 ∼ 9:55
Osamu Saeki（Kyushu University, IMI)

10:00 ∼ 11:00
Michihisa Wakui（Kansai University）
A new algorithm of the normalized Jones polynomials of rational links

11:20 ∼ 12:20
Kimihiko Motegi（Nihon University） Online
The Strong Slope Conjecture for Mazur pattern satellite knots

14:00 ∼ 15:00
Toshitake Kohno（Meiji University/The University of Tokyo) Online
Temperley-Lieb-Jones category and the space of conformal blocks

15:20 ∼ 16:40 To the memory of Professor Toshie Takata:
Haruko Takayama (Nishi)（Josai University) Online
Tomoyoshi Ibukiyama (Osaka University) Online
and some others

17:00 ∼ 18:00
Christian Kassel（Université de Strasbourg/CNRS）Online
Braid groups and Steinberg groups

March 16th (Wedes)

10:00 ∼ 11:00
Jun Murakami（Waseda University）
Volume potential function and its applications

11:20 ∼ 12:20
Yoshiyuki Yokota（Tokyo Metropolitan University）
On Neumann-Zagier matrices and generalized angle structures for hyperbolic knots

14:00 ∼ 15:00
Kazuo Habiro（Kyoto University）
Ribbon Yetter-Drinfeld modules and tangle invariants

15:20 ∼ 16:20 To the memory of Professor Toshie Takata:
Sakie Suzuki（Tokyo Institute of Technology) Online
and some others

16:40 ∼ 17:40
Hitoshi Murakami（Tohoku University） Online
The colored Jones polynomial of the figure-eight knot

March 17th (Thurs)

10:00 ∼ 11:00
Eiko Kin（Osaka University）
Braids and fibered double branched covers of 3-manifolds

11:20 ∼ 12:20
Hidekazu Furusho（Nagoya University）
Kashiwara-Vergne Lie algebra and Goncharov's dihedral Lie algebra in mould theory

14:00 ∼ 15:00
Hisatoshi Kodani（Tohoku University） Online
On partial generalization of Hodge correlators associated with diagrams al-

lowed to have loops

15:20 ∼ 15:50
Densuke Shiraishi（Osaka University）
On functional equations of ℓ-adic Galois polylogarithms

16:10 ∼ 16:40
Hikaru Hirano（Kyushu University）
Brylinski–McLaughlin's state space and its arithmetic analogue

March 18th（Fri）

10:00 ∼ 11:00
Hiroyuki Ochiai（Kyushu University, IMI）
On arithmetic-geometric means and hypergeometric functions contained in Gauss' *Werke*

11:20 ∼ 12:20
Takeo Uramoto（Kyushu University, IMI）
Classical class field theory meets algebraic language theory

14:00 ∼ 14:30
Ryoto Tange（Waseda University） Online
On adjoint homological Selmer modules for SL(2)-representations of knot groups

14:50 ∼ 15:20
Hyuga Yoshizaki（Tokyo University of Science） Online
Weber's class number problem for cyclic covers of knots

15:40 ∼ 16:40
Toshiki Matsusaka（Nagoya University）
Modular transformations of homological blocks for Seifert fibered homology 3-spheres

Abstract

March 15th (Tues)

Michihisa Wakui (Kansai University)
A new algorithm of the normalized Jones polynomials of rational links

In connection with cluster algebras, snake graphs and q-integers, Kyungyong Lee and Ralf Schiffler recently found a formula for computing the (normalized) Jones polynomials of rational links in terms of continued fraction expansion of rational numbers. Sophie Morier-Genoud and Valentin Ovsienko introduced q-deformed continued fractions, and showed that by using them each coefficient of the normalized Jones polynomial counted quiver representations of type A_n. In this talk we introduce q-deformed integers defined by coprime pairs of natural numbers, which are motivated by the denominators and the numerators of their q-deformed continued fractions, and give an efficient algorithm for computing the normalized Jones polynomials of rational links.

Kimihiko Motegi (Nihon University)
The Strong Slope Conjecture for Mazur pattern satellite knots

The Slope Conjecture proposed by Garoufalidis asserts that the degree of the colored Jones polynomial determines a boundary slope, and its refinement, the Strong Slope Conjecture proposed by Kalfagianni and Tran asserts that the linear term in the degree determines the topology of an essential surface that satisfies the Slope Conjecture. Under certain hypotheses, we show that a Mazur pattern satellite knots satisfy the Strong Slope Conjecture if the original knot does. Consequently, combining with previous results, any knot obtained by a finite sequence of cabling, connected sums, Whitehead doubling and taking Mazur pattern satellites of adequate knots (including alternating knots) or torus knots satisfies the Strong Slope Conjecture. This is joint work with Kenneth L. Baker (University of Miami) and Toshie Takata.

Toshitake Kohno (Meiji University/The University of Tokyo)
Temperley-Lieb-Jones category and the space of conformal blocks

We show that the set of morphisms of the colored Temperley-Lieb-Jones category at roots of unity is isomorphic to the space of conformal blocks in the Wess-Zumino-Witten conformal field theory. We describe the braid group action on these spaces and explain that the above isomorphism is equivariant and that the representations are unitary and irreducible. To prove this we use an expression of the space of conformal blocks by multi-dimensional hypergeometric integrals. This investigation leads us to reveal a family of unitary and irreducible representations of the braid groups contained in homological representations at roots of unity.

Christian Kassel（Université de Strasbourg/CNRS）
Braid groups and Steinberg groups

We construct a homomorphism from the braid group on $2n + 2$ strands to the Steinberg group associated with the Lie type C_n and with integer coefficients. This homomorphism lifts the well-known symplectic representation of the braid groups. We shall describe its image, expressing it in terms of the level 2 congruence subgroup of the symplectic modular group. We shall also determine its kernel. This is joint work with François Digne (Amiens).

March 16th (Wedes)

Jun Murakami（Waseda University）
Volume potential function and its applications

The volume potential function appears as a certain limit of the colored Jones polynomial in the study of the volume conjecture. This volume potential function is conjectured to relate the A-polynomial and the twisted Reidemeister torsion. The relation to the A-polynomial is studied as AJ-conjecture, and the relation to the twisted Reidemeister torsion is conjectured by Gkukov-H.Murakami and is proved for two-bridge knots by Ohtsuki-Takata. In this talk, such relations are explained for twisted Whitehead links and some other examples. This is a joint with A. Tran.

Yoshiyuki Yokota（Tokyo Metropolitan University）
On Neumann-Zagier matrices and generalized angle structures for hyperbolic knots

We first review the simplectic property of the Neumann-Zagier matrix obtained from an ideal triangulation of a hyperbolic knot complement M. We then give a nice parametrization of the generalized angle structures, assignments of real numbers to the pairs of opposite edges of the tetrahedra, of M, and show that the volume of M is given by a critical value of the volume functional defined on such parametrized space.

Kazuo Habiro (Kyoto University)
Ribbon Yetter-Drinfeld modules and tangle invariants

It is well known that finite-dimensional Yetter-Drinfeld modules over a Hopf algebra form a braided, pivotal category, but not necessarily a ribbon category. We introduce a notion of ribbon Yetter-Drinfeld modules and show that finite-dimensional ribbon Yetter-Drinfeld modules form a ribbon category, and therefore give rise to tangle invariants. This is joint work with Yuka Kotorii.

Hitoshi Murakami (Tohoku University)
The colored Jones polynomial of the figure-eight knot

For any positive integer N, the colored Jones polynomial of a knot gives a Laurent polynomial in q. When we replace q with $\exp(u/N)$ for a fixed complex number u, we have a function $f_u(N)$ from positive integers to complex numbers. I will talk about relations between the asymptotic behaviors of $f_u(N)$ of the figure-eight knot for various u, and representations of the fundamental group of the knot complement to $\mathrm{SL}(2;\mathbb{C})$. Part of this work is a joint work with Anh Tran.

March 17th (Thurs)

Eiko Kin (Osaka University)
Braids and fibered double branched covers of 3-manifolds

The branched virtual fibering theorem by Sakuma states that every closed orientable 3-manifold M with a Heegaard surface of genus g has a branched double cover which is a genus g surface bundle over the circle. It is proved by Brooks that such a surface bundle can be chosen to be hyperbolic. In a previous result with Susumu Hirose, we proved that when M is the 3-sphere S^3,

the minimal entropy over all hyperbolic, genus g surface bundles as branched double covers of S^3 behaves like $1/g$. In this talk, I will explain that infinitely many closed 3-manifolds have the same property. To prove this claim, we use an alternative construction of surface bundles in Sakuma's theorem when M is a branched double cover of S^3 branched over a link. A feature of surface bundles coming from our construction is that the monodromies can be read off the braids obtained from the links as the branched set. This is a joint work with Susumu Hirose.

Hidekazu Furusho (Nagoya University)
Kashiwara-Vergne Lie algebra and Goncharov's dihedral Lie algebra in mould theory

We introduce the Kashiwara-Vergne bigraded Lie algebra associated with a finite abelian group. By giving its mould theoretic interpretation, we show that it includes the Goncharov's dihedral Lie algebra, which generalizes the result of Raphael and Schneps. It is a joint work with Nao Komiyama.

Hisatoshi Kodani（Tohoku University）
On partial generalization of Hodge correlators associated with diagrams allowed to have loops

Goncharov's Hodge correlators are complex numbers given as certain integrals assigned to Riemann surfaces. They contain a wide variety of arithmetic invariants such as classical (elliptic) polylogarithms and special values of some L-functions, and provide an alternative way to describe the standard real mixed Hodge structure on the nilpotent completion of the fundamental group of a punctured Riemann surface given by Chen's iterated integral. As he showed, the Hodge correlators can be thought of as tree diagrams part of large N asymptotic expansion of a matrix model. However, the structure and properties of loop diagrams part was left as problem and has been still unknown, since the loop part could be divergent. In this talk, we explain a partial generalization of Hodge correlators associated with diagrams allowed to have loops and their several propeties. This talk is based on a joint work with Y. Terashima.

Densuke Shiraishi (Osaka University)
On functional equations of ℓ-adic Galois polylogarithms

The ℓ-adic Galois polylogarithm is an arithmetic function on the absolute Galois group with values in ℓ-adic numbers, which is defined as a certain coefficient of the ℓ-adic associator arising from the Galois action on the etale fundamental groupoid of the projective line minus three points. This function was introduced by Wojtkowiak as an ℓ-adic Galois analogue of the classical polylogarithm, and typical functional equations of ℓ-adic Galois dilogarithms are proved by Nakamura and Wojtkowiak. In this talk, we discuss some new functional equations for ℓ-adic Galois polylogarithms that are not found in previous works.

Hikaru Hirano (Kyushu University)
Brylinski–McLaughlin's state space and its arithmetic analogue

Brylinski and McLaughlin introduced a certain state space as a variant of the space of conformal blocks in (2+1)-dim Dijkgraaf–Witten TQFT, and posed a question on their space. We give an answer to it and study an arithmetic analogue of the state space.

March 18th（Fri）

Hiroyuki Ochiai（Kyushu University, IMI）
On arithmetic-geometric means and hypergeometric functions contained in Gauss' *Werke*

There are contained some numerical examples of arithmetic-geometric means in Gauss' *Werke*. I will talk about my discovery on those numerical values. It is known that arithmetic-geometric means can be expressed by the special values of hypergeometric functions. I will also talk about my observation on some family of special functions related to those hypergeometric functions.

Takeo Uramoto（Kyushu University, IMI）
Classical class field theory meets algebraic language theory

Algebraic language theory is a subfield of the theory of computation, and since the 1960s, has developed semigroup-theoretic methodology for solving several logical/combinatorial decision problems concerning regular languages (= sets of finite words accepted by finite automata). In our recent works, we

observed that algebraic language theory can be unified with galois theory in a certain precise categorical sense, and more importantly, that this unification sheds a new light on classical class field theory and complex multiplication. In this talk, we will overview these recent results and discuss some future perspective.

Ryoto Tange（Waseda University）
On adjoint homological Selmer modules for SL(2)-representations of knot groups （Joint work with T. Kitayama, M. Morishita, Y. Terashima）

We introduce the adjoint homological Selmer module for an SL(2)-representation of a knot group, which may be seen as an analogue of the adjoint Selmer module for a Galois representation in number theory. This is joint work with Takahiro Kitayama, Masanori Morishita, and Yuji Terashima.

Hyuga Yoshizaki (Tokyo University of Science)

We study an analogue of Weber's class number problem for cyclic covers of knots in the spirit of arithmetic topology. Let p be a prime number. Weber's problem is an unsolved problem with a long history in number theory asking the class numbers of certain cyclic p^n-th extensions of the rationals $\mathbb{Q}$. Since the analogue of the class numbers of number fields is the sizes of the first homology groups of 3-manifolds, we study the sizes of the first homology groups of cyclic p^n-th covers of knots in S^3. In number theory side, the speaker previously proved the p-adic convergence of the class numbers, but their limit value is not yet known for any example. In this talk, on the other hand, we prove an analogous result in knot theory side and concretely calculate their limit for the torus knots and the twist knots. This talk is based on a joint work with Jun Ueki (Tokyo Denki University).

Toshiki Matsusaka（Nagoya University）
Modular transformations of homological blocks for Seifert fibered homology 3-spheres

Recently, Gukov-Pei-Putrov-Vafa introduced q-series-valued invariants called homological blocks for any plumbed 3-manifolds. In this talk, for any Seifert fibered homology 3-sphere, we give modular transformation formulas of homological blocks. Moreover, based on the modular transformation formulas,

we have explicit asymptotic expansion formulas for the Witten-Reshetikhin-Turaev invariants, which give a new proof of a version by Andersen of the Witten asymptotic conjecture. This is joint work with Yuji Terashima (Tohoku University).